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CP violation in $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ decay

Göran Fäldt[1]

Division of Nuclear Physics, Box 535, 751 21 Uppsala, Sweden

Evgeny Shabalin[2]

Institute for Theoretical and Experimental Physics, 117218 Moscow, B.Cheremushkinskaya 25, Russia

Abstract

CP violation leads to a difference between the parameters g^+ and g^- describing the energy distributions of the charged pions produced in the $K^+ \to \pi^0\pi^0\pi^+$ and $K^- \to \pi^0\pi^0\pi^-$ decays. We study the difference $(g^+ - g^-)$ as a function of the relative contributions from the QCD-penguin and the electroweak-penguin diagrams. We find that the combination of these contributions in $(g^+ - g^-)$ is very similar to the corresponding one defining the parameter ε' in the $K_L \to 2\pi$ decays. This observation allows a determination of the value of $(g^+ - g^-)$, using data on ε'.

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[1]Electronic address: faldt@tsl.uu.se
[2]Electronic address: shabalin@heron.itep.ru

1 Introduction

Since 1989 it has been known that the magnitude of direct CP violation in the $K_L \to 2\pi$ decays crucially depends on the relative strengths of the imaginary parts of the QCD-penguin (QCDP) and the electroweak-penguin (EWP) contributions to the amplitude [1]. The reason for this sensitivity is that the contributions to ε' from the two diagrams have opposite signs and partially cancel one another.

As the dynamical structures of the amplitudes for $K^\pm \to (3\pi)^\pm$ differ from those for $K_L \to 2\pi$, there is no immediate relation between the strengths of direct CP violation in the $K_L \to 2\pi$ and the $K^\pm \to (3\pi)^\pm$ decays. In particular, it was observed in refs. [2, 3] that in contrast to the situation in $K_L \to 2\pi$, the CP violating effect in $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ produced by the QCDP contribution is enhanced by the EWP contribution.

However, in the present note, we shall demonstrate that the $K^\pm \to \pi^0\pi^0\pi^\pm$ decays are similar to the $K_L \to 2\pi$ decay in that the EWP contribution cancels part of the QCDP contribution. Due to this circumstance, we suggest that a simultaneous study of the decays $K^\pm \to \pi^0\pi^0\pi^\pm$ and $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ could throw new light on the relative strengths of the QCDP and the EWP mechanisms in direct CP violation.

We shall estimate, in the framework of the Standard Model, the CP violating contributions to the slope parameters g^+ and g^- characterizing the charged pion energy distributions in the $K^\pm \to \pi^0\pi^0\pi^\pm$ decays (formerly τ' decay). The slope parameters are defined by the expansion

$$|M(K^\pm(k) \to \pi^0(p_1)\pi^0(p_2)\pi^\pm(p_3))|^2 \propto 1 + g^\pm Y + ..., \tag{1.1}$$

where

$$Y = (s_3 - s_0)/m_\pi^2, \quad s_i = (k - p_i)^2, \quad s_0 = m_K^2/3 + m_\pi^2. \tag{1.2}$$

Our tenet is corroborated by a calculation of the amplitudes to leading non-vanishing order in a momentum expansion. As was previously found for the $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ decays [2, 3], higher-order corrections do not considerably change

the conclusion concerning the relative magnitudes of the QCDP and EWP contributions to the difference $(g^+ - g^-)_\tau$. The role of higher-order corrections in the τ' decays will be considered elsewhere.

2 The $K^\pm \to \pi^0\pi^0\pi^\pm$ amplitude

Our starting point is the $\Delta S = 1$ effective non-leptonic Lagrangian proposed in ref. [4]

$$L(\Delta S = 1) = \sqrt{2} G_{\mathrm{F}} \sin\theta_{\mathrm{C}} \cos\theta_{\mathrm{C}} \sum_i c_i O_i \,, \tag{2.3}$$

where O_{1-6} are effective four-quark operators represented by the operator products

$$O_1 = \bar{s}_L\gamma_\mu d_L \cdot \bar{u}_L\gamma_\mu u_L - \bar{s}_L\gamma_\mu u_L \cdot \bar{u}_L\gamma_\mu d_L \ ; \qquad (\{8_f\}, \Delta I = 1/2), \tag{2.4}$$

$$\begin{aligned} O_2 =& \bar{s}_L\gamma_\mu d_L \cdot \bar{u}_L\gamma_\mu u_L + \bar{s}_L\gamma_\mu u_L \cdot \bar{u}_L\gamma_\mu d_L + 2\bar{s}_L\gamma_\mu d_L \cdot \bar{d}_L\gamma_\mu d_L \\ &+ 2\bar{s}_L\gamma_\mu d_L \cdot \bar{s}_L\gamma_\mu s_L \ ; \qquad (\{8_d\}, \Delta I = 1/2), \end{aligned} \tag{2.5}$$

$$\begin{aligned} O_3 =& \bar{s}_L\gamma_\mu d_L \cdot \bar{u}_L\gamma_\mu u_L + \bar{s}_L\gamma_\mu u_L \cdot \bar{u}_L\gamma_\mu d_L + 2\bar{s}_L\gamma_\mu d_L \cdot \bar{d}_L\gamma_\mu d_L \\ &- 3\bar{s}_L\gamma_\mu d_L \cdot \bar{s}_L\gamma_\mu s_L \ ; \qquad (\{27\}, \Delta I = 1/2), \end{aligned} \tag{2.6}$$

$$\begin{aligned} O_4 =& \bar{s}_L\gamma_\mu d_L \cdot \bar{u}_L\gamma_\mu u_L + \bar{s}_L\gamma_\mu u_L \cdot \bar{u}_L\gamma_\mu d_L \\ &- \bar{s}_L\gamma_\mu d_L \cdot \bar{d}_L\gamma_\mu d_L \ ; \qquad (\{27\}, \Delta I = 3/2), \end{aligned} \tag{2.7}$$

$$O_5 = \bar{s}_L\gamma_\mu\lambda^a d_L(\sum_{q=u,d,s} \bar{q}_R\gamma_\mu\lambda^a q_R) \ ; \qquad (\{8\}, \Delta I = 1/2), \tag{2.8}$$

$$O_6 = \bar{s}_L\gamma_\mu d_L(\sum_{q=u,d,s} \bar{q}_R\gamma_\mu q_R) \ ; \qquad (\{8\}, \Delta I = 1/2) \ . \tag{2.9}$$

Among these operators, only O_4 generates $\Delta I = 3/2$ transitions. The operators $O_{5,6}$ originate from the QCDP diagrams. To calculate CP-violating effects, also the operators $O_{7,8}$ generated by the EWP diagrams must be added,

$$O_7 = \frac{3}{2}\bar{s}\gamma_\mu(1+\gamma_5)d(\sum_{q=u,d,s} e_q\bar{q}\gamma_\mu(1-\gamma_5)q); \qquad (\Delta I = 1/2, 3/2), \tag{2.10}$$

$$O_8 = -12\sum_{q=u,d,s} e_q(\bar{s}_L q_R)(\bar{q}_R d_L); \quad e_q = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}); \quad (\Delta I = 1/2, 3/2). \tag{2.11}$$

The corresponding Wilson coefficients $c_{7,8}$ are small, being proportional to α_{em}. The coefficients c_{5-8} contain the imaginary parts necessary for CP violation.

Bosonization of the operators O_i is achieved through the substitutions [5]

$$\bar{q}_j(1+\gamma_5)q_k = -\frac{1}{\sqrt{2}}F_\pi r(U - \frac{1}{\Lambda^2}\partial^2 U)_{kj} \ , \tag{2.12}$$

$$\bar{q}_j\gamma_\mu(1+\gamma_5)q_k = i[(\partial_\mu U)U^\dagger - U(\partial_\mu U^\dagger) - \frac{rF_\pi}{\sqrt{2}\Lambda^2}(m(\partial_\mu U^\dagger) - (\partial_\mu U)m)]_{kj} \ . \tag{2.13}$$

Here, m is the diagonal quark-mass matrix,

$$m = \mathrm{Diag}\{m_u, m_d, m_s\} \ ,$$

and the remaining parameters are defined as

$$r = 2m_\pi^2/(m_u+m_d), \quad \Lambda \approx 1 \text{ GeV}, \quad F_\pi = 93 \text{ MeV}.$$

The 3×3 U-matrix is written as an expansion

$$U = \frac{F_\pi}{\sqrt{2}}\left(1 + \frac{i\sqrt{2}\hat{\pi}}{F_\pi} - \frac{\hat{\pi}^2}{F_\pi^2} + a_3\left(\frac{i\hat{\pi}}{\sqrt{2}F_\pi}\right)^3 + 2(a_3-1)\left(\frac{i\hat{\pi}}{\sqrt{2}F_\pi}\right)^4 + ...\right) \tag{2.14}$$

in the pseudoscalar nonet-meson-field matrix $\hat{\pi}$

$$\hat{\pi} = \begin{pmatrix} \frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} + \frac{\pi_3}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} - \frac{\pi_3}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \frac{\pi_0}{\sqrt{3}} - \frac{2\pi_8}{\sqrt{6}} \end{pmatrix} . \tag{2.15}$$

The PCAC condition demands $a_3 = 0$ [6] and we adopt this condition as well, bearing in mind, that on mass shell, the values of the mesonic amplitudes are independent of the parameter a_3.

In the calculation of the $K \to 3\pi$ amplitudes we make use of the Fierz identities for the colour matrices

$$\begin{aligned} \delta^\alpha_\beta \delta^\gamma_\delta &= \tfrac{1}{3}\delta^\alpha_\delta \delta^\gamma_\beta + \tfrac{1}{2}\lambda^\alpha_\delta \lambda^\gamma_\beta \\ \lambda^\alpha_\beta \lambda^\gamma_\delta &= \tfrac{16}{9}\delta^\alpha_\delta \delta^\gamma_\beta - \tfrac{1}{3}\lambda^\alpha_\delta \lambda^\gamma_\beta \end{aligned}$$

as well as the Fierz identities for the Dirac matrices

$$\bar{s}\gamma_\mu(1+\gamma_5)d\cdot\bar{q}\gamma_\mu(1-\gamma_5)q = -2\bar{s}(1-\gamma_5)q\cdot\bar{q}(1+\gamma_5)d\ .$$

Thus, in leading order non-vanishing approximation our result for the matrix element can be expressed as

$$\begin{aligned} M\left(K^\pm \to \pi^0(p_1)\pi^0(p_2)\pi^\pm(p_3)\right) = \\ = \kappa\left[1 \pm ia_{KM} + \frac{3m_\pi^2}{m_K^2}\left(1-\frac{9c_4}{2c_0}\right)Y(1\pm ib_{KM}) + ...\right]\ . \end{aligned} \tag{2.16}$$

The kinematic variable Y is defined in eq.(1.2). The overall strength is regulated by the parameter

$$\kappa = \frac{G_F m_K^2}{6\sqrt{2}} c_0 \sin\theta_C \cos\theta_C\ , \tag{2.17}$$

and the remaining parameters are functions of the following combinations

$$c_0 = c_1 - c_2 - c_3 - c_4 + \frac{32}{9}\beta \mathrm{Re}\tilde{c}_5\ , \tag{2.18}$$

$$\tilde{c}_5 = c_5 + \frac{3}{16}c_6, \quad \tilde{c}_7 = c_7 + 3c_8\ , \tag{2.19}$$

$$\beta = \frac{2m_\pi^4}{\Lambda^2(m_u+m_d)^2}\ . \tag{2.20}$$

The terms a_{KM} and b_{KM} in the amplitude (2.16) are the imaginary parts of the amplitude generated by the Kobayashi-Maskawa phase. Explicitely, we find

$$a_{KM} = \beta\left(\frac{32}{9}\mathrm{Im}\tilde{c}_5 + \frac{6\Lambda^2\mathrm{Im}\tilde{c}_7}{m_K^2}\right)/c_0 \tag{2.21}$$

$$b_{KM} = \beta\left(\frac{32}{9}\mathrm{Im}\tilde{c}_5 + \frac{3\Lambda^2\mathrm{Im}\tilde{c}_7}{m_K^2 - m_\pi^2}\right)/(c_0 - \frac{9c_4}{2})\ , \tag{2.22}$$

with coefficients as above.

Our approach can also be used to caculate the $K \to 2\pi$ amplitudes. For their real parts we get

$$M(K_1^0 \to \pi^+\pi^-) = \frac{G_F F_\pi}{\sqrt{2}}\sin\theta_C\cos\theta_C(m_K^2 - m_\pi^2)c_0\ , \tag{2.23}$$

$$M(K^+ \to \pi^+\pi^0) = \frac{G_F F_\pi}{\sqrt{2}}\sin\theta_C\cos\theta_C(m_K^2 - m_\pi^2)(\tfrac{3}{2}c_4)\ . \tag{2.24}$$

A comparison between the real parts of the amplitudes of eqs (2.16) and (2.23) shows that their ratio is nothing more than a reflection of the well-known relation

$$\begin{aligned} M\left(K^+(k) \to \pi^0(p_1)\pi^0(p_2)\pi^+(p_3)\right) = \frac{1}{6F_\pi} M(K_1^0 \to \pi^+\pi^-) \\ \times \left[1 + \frac{3m_\pi^2}{m_K^2}\left(1 - \frac{3M(K^+ \to \pi^+\pi^0)}{M(K_1^0 \to \pi^+\pi^-)}\right) Y\right] \end{aligned} \tag{2.25}$$

obtained earlier [7, 8, 9][3] using soft-pion techniques and current algebra.

From the data on the $K \to 2\pi$ decay rates [10], it follows that

$$c_1 - c_2 - c_3 + \frac{32}{9}\beta\,\mathrm{Re}\tilde{c}_5 = -10.13\ , \tag{2.26}$$

$$c_4 = 0.328\ . \tag{2.27}$$

Furthermore, the combination $\beta\,\mathrm{Re}\tilde{c}_5$ can be determined separately, provided we are willing to accept the estimate of Shifman *et al.* [4, 11],

$$c_1 - c_2 - c_3 = -2.89\ , \tag{2.28}$$

which leads to the value

$$\frac{32}{9}\beta\,\mathrm{Re}\tilde{c}_5 = -7.24\ . \tag{2.29}$$

To estimate CP-odd effects in $K^\pm \to (3\pi)^\pm$ decays, described by the coefficients a_{KM} and by b_{KM}, we need a certain combination of the parameters $\mathrm{Im}\tilde{c}_5$ and $\mathrm{Im}\tilde{c}_7$. The theoretical preditions for these parameters are very uncertain and different authors (see [3]) give different results. Fortunately, the combination entering the $K^\pm \to \pi^0\pi^0\pi^\pm$ amplitude turns out to be similar to the combination determining the parameter ε' in $K_L \to 2\pi$ decay [2]. This circumstance allows us to obtain a reliable estimate of $(g^+ - g^-)_{\tau'}$.

3 Estimate of the CP-odd difference $(g^+ - g^-)_{\tau'}$

Although the amplitude (2.16) incorporates the imaginary terms necessary for CP violation, this is not sufficient for producing observable CP-violating effects.

[3] In [9] there is a misprint: a factor y is missing after the factor $(1 + \frac{3\delta}{1+\theta})$ in the expression for the $K^+ \to \pi^0\pi^0\pi^+$ amplitude, eq.(6.9).

In fact, the observable effects arise from the interference between these CP-odd terms and the CP-even imaginary terms created by the strong-interaction final-state rescattering between the pions. The strong-interaction effects are introduced into the $K \to 3\pi$ amplitudes of eq.(2.16) by adding two terms, $a_{\tau'}$ and $b_{\tau'}$, so that

$$\begin{aligned} M\left(K^{\pm}(k) \to \pi^0(p_1)\pi^0(p_2)\pi^{\pm}(p_3)\right) = \kappa \frac{1 \pm i a_{KM}}{(1+a_{KM}^2)^{1/2}}[1 + i a_{\tau'} + \\ + \frac{3m_\pi^2}{m_K^2}\left(1 - \frac{9c_4}{2c_0}\right) Y(1 + i b_{\tau'} \pm i(b_{KM} - a_{KM})) + ...] \ . \end{aligned} \tag{3.30}$$

This assumption is valid as long as the rescattering contribution can be treated in the linear approximation.

The slope parameters g^+ and g^- were defined in eq.(1.1). From this definition and eq.(3.30) we get for the relative difference in slope parameter for the $K \to 3\pi$ decays

$$\Delta g_{\tau'} = \left(\frac{g^+ - g^-}{g^+ + g^-}\right)_{\tau'} = \frac{a_{\tau'}(b_{KM} - a_{KM})_{\tau'}}{1 + a_{\tau'} b_{\tau'}} \tag{3.31}$$

The strong-interaction-rescattering parameters, $a_{\tau'}$ and $b_{\tau'}$, are determined by calculating the imaginary parts of the loop diagrams of Fig. 1. Putting the intermediate pions on shell (see Appendix) yields, in leading approximation,

$$a_{\tau'} = 0.12, \qquad b_{\tau'} = 0.49. \tag{3.32}$$

The CP-odd numerator of eq.(3.31) can be calculated from the expressions in eqs (2.21) and (2.22), and is found being equal to

$$\begin{aligned} (b_{KM} - a_{KM})_{\tau'} &= \frac{16 c_4 \beta \ \mathrm{Im}\tilde{c}_5}{c_0(c_0 - \frac{9}{2}c_4)} - \frac{6\beta\Lambda^2 \ \mathrm{Im}\tilde{c}_7}{m_K^2 c_0}\left(1 - \frac{c_0 m_K^2}{2(m_K^2 - m_\pi^2)(c_0 - \frac{9}{2}c_4)}\right) \\ &= 0.042\beta \, \mathrm{Im}\tilde{c}_5 \left(1 + 27.8 \ \mathrm{Im}\tilde{c}_7/\mathrm{Im}\tilde{c}_5\right) \ . \end{aligned} \tag{3.33}$$

The combination of Wilson coefficients in this formula,

$$\beta \ \mathrm{Im}\tilde{c}_5 \left(1 + 27.8 \ \mathrm{Im}\tilde{c}_7/\mathrm{Im}\tilde{c}_5\right) \ , \tag{3.34}$$

is very similar to another combination

$$\beta\,\mathrm{Im}\tilde{c}_5(1+\frac{24.36}{1-\Omega}\cdot\frac{\mathrm{Im}\tilde{c}_7}{\mathrm{Im}\tilde{c}_5}) = -\frac{(1.63\pm 0.25)\cdot 10^{-4}}{1-\Omega}\beta\,\mathrm{Re}\tilde{c}_5 \qquad (3.35)$$

defining the direct CP-violating parameter ε' in $K_L \to 2\pi$ decay [2, 3]. The parameter Ω takes into account isospin-breaking contributions generated by the two-step transition $K^0 \to \pi^0\eta(\eta') \to \pi^0\pi^0$.

At $\Omega = 0.124$ expressions (3.34) and (3.35) coincide, giving

$$\Delta g_{\tau'} = (1.8 \pm 0.28)\cdot 10^{-6}\ , \qquad (3.36)$$

and at Ω=0.25

$$\Delta g_{\tau'} = 2.1\cdot 10^{-6}(1-\frac{4.7\,\mathrm{Im}\tilde{c}_7/\mathrm{Im}\tilde{c}_5}{1+32.48\,\mathrm{Im}\tilde{c}_7/\mathrm{Im}\tilde{c}_5})\ . \qquad (3.37)$$

Both values of Ω are in line with estimates figurating in the literature (see [12] and references therein).

4 The CP-odd difference $(g^+ - g^-)_\tau$

As we shall now show, our result for the slope-parameter difference in τ' decay, as embodied in eq.(3.33), enables us to draw quite precise conclusions concerning the magnitude of the slope-parameter difference in another decay, namely the $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ decay, or τ decay. From eqs (33) and (34) in ref. [3], we derive the following relation

$$(b_{KM} - a_{KM})_\tau = -2\left[\frac{16c_4\beta\,\mathrm{Im}\tilde{c}_5}{c_0(c_0+9c_4)} + \frac{3\beta\Lambda^2\,\mathrm{Im}\tilde{c}_7}{m_K^2 c_0}\left(1+\frac{12c_4 m_K^2}{\Lambda^2(c_0+9c_4)}\right)\right]\ . \qquad (4.38)$$

The slope-parameter difference Δg_τ is again given by expression (3.31), provided index τ' is everywhere replaced by τ. The value of the rescattering parameter a does not change, but that of b does,

$$a_\tau = 0.12, \qquad b_\tau = 0.714. \qquad (4.39)$$

Combining eqs (3.31), (3.33) and (4.38) we can form the ratio of the slope parameters differences,

$$\frac{-\Delta g_\tau}{\Delta g_{\tau'}} = 2\frac{c_0 - 9c_4/2}{c_0+9c_4}\cdot\frac{1-14.34\,\mathrm{Im}\tilde{c}_7/\mathrm{Im}\tilde{c}_5}{1+27.8\,\mathrm{Im}\tilde{c}_7/\mathrm{Im}\tilde{c}_5}\cdot\frac{1+a_{\tau'}b_{\tau'}}{1+a_\tau b_\tau}\ . \qquad (4.40)$$

Now, only negative values of the ratio $\mathrm{Im}\tilde{c}_7/\mathrm{Im}\tilde{c}_5$ appear in the literature [3]. If furthermore, we assume that the numerical value of this ratio is so small that the sign of the right hand side of eq.(4.40) is positive, we may conclude that

$$-\Delta g_\tau \geq 3.1\Delta g_{\tau'} > 0.56 \cdot 10^{-5}. \tag{4.41}$$

This result differs from other estimates, as exemplified by refs. [13] and [14]

$$-\Delta g_\tau = 1.8\Delta g_{\tau'} \quad [13], \qquad -\Delta g_\tau = 2.2\Delta g_{\tau'} \quad [14]. \tag{4.42}$$

Moreover, as follows from our discussion in Sect. 3, we strongly believe $\Delta g_{\tau'}$ to be of order 10^{-6}. In contrast, Δg_τ can reach values of order 10^{-5} , providing the EWP contribution cancels out a considerable part of the QCDP contribution (see eq.(4.40)). For example, if the EWP cancels half of the QCDP contribution, then

$$-\Delta g_\tau = 7.8\Delta g_{\tau'} \geq 1.4 \cdot 10^{-5} \tag{4.43}$$

and if the EWP cancels three-quarters of the QCDP contribution, then

$$-\Delta g_\tau = 17.2\Delta g_{\tau'} \geq 3.1 \cdot 10^{-5}. \tag{4.44}$$

Δg_τ (in units 10^{-5})	$\Delta g_{\tau'}$ (in units 10^{-5})	Refs.
-700 ± 500	-15 ± 275	[15]
$\lvert\Delta g_\tau\rvert_{LO} \leq 0.7$	-	[16]
-0.16	-	[17]
$\lvert\Delta g_\tau\rvert = 38.2$	$\lvert\Delta g_{\tau'}\rvert = 31.5$	[18]
-0.23 ± 0.06	0.13 ± 0.04	[13]
$(-4.9 \pm 0.9)\sin\delta$	-	[2]
-2.4 ± 1.2	1.1 ± 0.7	[14]
$-(3.0 \pm 0.5)x; \quad 0.5 < x < 5.0$	-	[3]
$(-\Delta g_\tau)_{LO} > (0.56 \pm 0.09)f(x)$ At $x = 1, \quad (-\Delta g_\tau) = 2.9 \pm 0.6$	0.18 ± 0.03	present work

Table 1: Values for the slope-parameter ratios Δg_τ and $\Delta g_{\tau'}$ in τ and τ' decays, in units of 10^{-5}.

These examples show that a simultaneous measurement of $\Delta g_{\tau'}$ and Δg_τ can clear up the question about the true relative strength of EWP and QCDP mechanisms in direct CP violation. The estimates of the values Δg_τ and $\Delta g_{\tau'}$ as obtained in other investigations are summarised in Table 1.

5 Concluding remarks

We have calculated the CP-odd difference of slope parameters, $\Delta g_{\tau'}$ of eq.(3.31), in the τ' decays $K^\pm \to \pi^0\pi^0\pi^\pm$ in leading non-vanishing approximation in a momentum expansion of the decay amplitude.

We observe that the difference of slope parameters $\Delta g_{\tau'}$ in $K^\pm \to \pi^0\pi^0\pi^\pm$ decay and the parameter ε' in $K_L \to 2\pi$ decay both depend practically on one and the same combination of the Wilson coefficients $\mathrm{Im}\tilde{c}_5$ and $\mathrm{Im}\tilde{c}_7$. This observation permits a reliable estimate of $\Delta g_{\tau'}$ using the known magnitude of ε'.

A comparison with the value of the corresponding parameter Δg_τ in the τ decays $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ shows that Δg_τ is expected to be at least 3 times larger than $\Delta g'_\tau$. In fact, it may be even one order of magnitude larger, provided there is a sizeable cancellation between the electroweak-penguin and the QCD-penguin contributions to the parameter ε'. Such a cancellation is not excluded [3, 19].

We have not considered the possibility of a sequential decay $K^\pm \to \pi^0\eta\pi^\pm \to \pi^0\pi^0\pi^\pm$ through an intermediate $\eta \to \pi^0$ transition, a correction which is of order p^4. We shall study this possibility elsewhere. In the case of $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ decay, higher-order corrections increase Δg_τ by 20%, but change very little the relation between electroweak-penguin and QCD-penguin contributions [3]. We expect a similar increase of $\Delta g_{\tau'}$ in the $K^\pm \to \pi^0\pi^0\pi^\pm$ decay, since $a_{\tau'} \approx a_\tau$, and according to ref.[3] p^4 corrections increase the value of a_τ by 30%.

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6 Appendix

Here, we shall calculate the CP-even imaginary part coming from the pion-rescattering diagrams displayed in fig. 1. The imaginary part of a diagram is obtained by cutting the internal lines as shown.

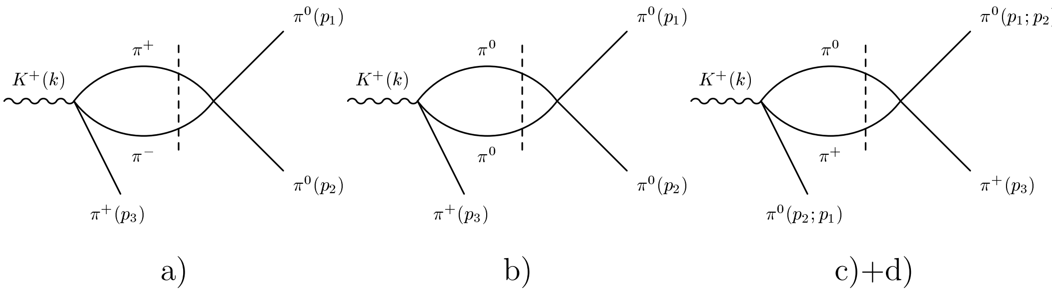

Figure 1: Rescattering diagrams for the imaginary part. Diagrams are cut along the dashed line. Diagrams c) and d) are related through $\pi^0(p_1) \leftrightarrow \pi^0(p_2)$.

We start from the amplitudes

$$M\left(K^+(k) \to \pi^0(p_1)\pi^0(p_2)\pi^+(p_3)\right) = A + B(s_0 - s_3) \tag{6.45}$$

$$M\left(K^+(k) \to \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)\right) = A' + B'(s_0 - s_3) \tag{6.46}$$

with kinematic variables as defined in eq.(1.2). The τ' decay amplitude, eq.(6.45), is given in eq.(2.16), of which we only need the leading real term. In the τ decay amplitude, eq.(6.46), the parameter A' is twice as large as A. For the $\pi\pi$ scattering amplitudes we insert the leading-order approximations,

$$M\left(\pi^+(q_1)\pi^-(q_2) \to \pi^0(q_3)\pi^0(q_4)\right) = \frac{i}{F_\pi^2}(s - m_\pi^2) \tag{6.47}$$

$$M\left(\pi^0(q_1)\pi^0(q_2) \to \pi^0(q_3)\pi^0(q_4)\right) = \frac{i}{F_\pi^2}(s + t + u - 3m_\pi^2) \tag{6.48}$$

$$M\left(\pi^0(q_1)\pi^+(q_2) \to \pi^0(q_3)\pi^+(q_4)\right) = \frac{i}{F_\pi^2}(t - m_\pi^2) \tag{6.49}$$

with $s = (q_1 + q_2)^2$, $t = (q_1 - q_3)^2$ and $u = (q_1 - q_4)^2$ as usual.

First, we calculate diagram a) with a $\pi^+\pi^-$ pair in the loop. The result is an imaginary contribution to the τ' decay

$$\delta M_a = \frac{i}{16\pi F_\pi^2}(s_3 - \mu^2)\sqrt{1 - \frac{4\mu^2}{s_3}}\left[A' + \tfrac{1}{2}B'(s_3 - s_0)\right] \ . \tag{6.50}$$

However, we are not interested in the exact value of δM_a. The slope parameters, eq.(1.1), are defined through an expansion in $Y = (s_3 - s_0)/m_\pi^2$. Moreover, we normalise the $K^+ \to \pi^+\pi^+\pi^-$ decay parameters as

$$\begin{aligned} A' &= \tfrac{2}{3} m_K^2 & (6.51) \\ B' &= 1 + 9c_4/c_0 = 0.718 \ , & (6.52) \end{aligned}$$

so that a short algebraic calculation gives as result

$$\delta M_a = i\frac{m_K^4}{72\pi F_\pi^2}\sqrt{\frac{s_0-4\mu^2}{s_0}}\left[1+\frac{3(s_3-s_0)}{m_K^2}\left\{1+\frac{2m_K^2 m_\pi^2}{3s_0(s_0-4m_\pi^2)}+\tfrac{1}{4}B'\right\}\right] \ . \tag{6.53}$$

Diagram b) with two neutral pions in the loop give a contribution to the imaginary part

$$\delta M_b = \frac{i}{32\pi F_\pi^2}\mu^2\sqrt{1-\frac{4\mu^2}{s_3}}\,[A + B(s_0 - s_3)] \ . \tag{6.54}$$

The parameters for the decay $K^+ \to \pi^+\pi^0\pi^0$ are

$$\begin{aligned} A &= \tfrac{1}{3} m_K^2 & (6.55) \\ B &= -(1 - 9c_4/2c_0) = -1.14 \ . & (6.56) \end{aligned}$$

The expansion of this contribution yields the result

$$\delta M_b = i\frac{m_K^2 m_\pi^2}{96\pi F_\pi^2}\sqrt{\frac{s_0-4\mu^2}{s_0}}\left[1+\frac{s_3-s_0}{m_K^2}\left\{\frac{2m_K^2 m_\pi^2}{3s_0(s_0-4m_\pi^2)} - 3B\right\}\right] \ . \tag{6.57}$$

There are two contributions, diagrams c) and d), with $\pi^+\pi^0$ in the loop, since the final state is symmetric in the two neutral pions, $\pi^0(p_1)$ and $\pi^0(p_2)$. We shall not give the exact expressions, corresponding to eqs (6.50) and (6.54), since they are somewhat complicated. The expansion of the sum of the two amplitudes results in an imaginary contribution

$$\begin{aligned} \delta M_c + \delta M_d =& i\frac{m_K^4}{144\pi F_\pi^2}\sqrt{\frac{s_0-4\mu^2}{s_0}}\left[-(1-\frac{3m_\pi^2}{m_K^2})\right. \\ &\left. + \frac{3(s_3-s_0)}{2m_K^2}\left\{1+\frac{2m_\pi^2(s_0-2m_\pi^2)}{s_0(s_0-4m_\pi^2)}+\frac{3m_\pi^2}{m_K^2}B\right\}\right] \ . \end{aligned} \tag{6.58}$$

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